A couple of additions

A sequence is called BOUNDED if it is bounded below AND bounded above.

Important Theorem: EVERY bounded & monotonic sequence has a limit,

only a floor: 1, 2, 3, 4, 5, ...

Only a ceiling: -1, -2, -3, -4, ...

converge - huh?

If a sequence has a limit, we say it converges

If a sequence has No limit, we say it diverges.

-1, 1, -1, 1, -1, 1, ...

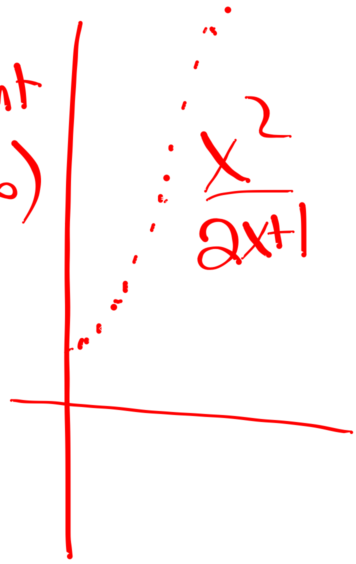
Feb 28-7:51 PM

$\left\{ \frac{1}{2^{n-1}} \right\}$ converges to a limit of 0

10.2/6

$$\left\{\frac{n^2}{2n+1}\right\} = \text{divergent } (\lim = +\infty)$$

$$\frac{1}{3}, \frac{4}{5}, \frac{9}{7}, \frac{16}{9}, \ldots$$



$$\lim_{x\to\infty} \frac{x^2}{2x+1} = \lim_{x\to\infty} \frac{x}{2+\frac{1}{x}} = \infty$$

$$\left\{ n\sin\left(\frac{\pi}{n}\right) \right\}_{n=1}^{\infty} \text{ converges?}$$

$$\lim_{n\to\infty} n\sin\left(\frac{\pi}{n}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{\pi}{n}\right)}{\frac{1}{n}} \overset{LH}{=}$$

$\infty \cdot 0$

if

$$\lim_{n\to\infty} \frac{\pi\cos\left(\frac{\pi}{n}\right)\left(-\frac{1}{n^2}\right)}{-\frac{1}{n^2}} = \pi$$

Series

Series are sums of the terms of a sequence.

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

We say that a series converges if the sequence of partial sums converges

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$

$S_4 = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$

$\vdots$

$S_1 = 1 - \frac{1}{2}$

$S_2 = 1 - \frac{1}{4}$

$S_3 = 1 - \frac{1}{8}$

$S_4 = 1 - \frac{1}{16}$

$\vdots$

$\left\{1 - \frac{1}{2^n}\right\}_{n=1}^{\infty}$

$$\lim_{n \to \infty} \left(1 - \frac{1}{2^n}\right) = 1$$

If the sequence of partial sums converges to a limit L, then we say the <u>sum</u> of the series is L.

Geometric Series

A geometric series is a series with the property that there is some r (called the common ratio) so that $a_{n+1} = (r)(a_n)$.

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

$$\left(r = \frac{1}{2}\right)$$

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

$$-\left(\frac{1}{2} S\right) = -\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots\right)$$

$$\frac{1}{2} S = \frac{1}{2} + 0 + 0 + 0 + 0 + 0 + \cdots$$

$$\frac{1}{2} S = \frac{1}{2}$$

$$\therefore S = 1$$

$$S = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \frac{3}{100000} + \cdots$$

$$.33333333333\ldots$$

$$-\frac{1}{10} S = -\left(\frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \frac{3}{100000} + \cdots\right)$$

$$\frac{9}{10} S = \frac{3}{10}$$

$$\therefore S = \frac{3}{10} \cdot \frac{10}{9} = \frac{3}{9} = \frac{1}{3}$$

A geometric series – generally – looks like

$$\sum_{n=1}^{\infty} ar^n = ar + ar^2 + ar^3 + \cdots$$

$$= a\sum_{n=1}^{\infty} r^n = a(r + r^2 + r^3 + \cdots)$$

$$S = r + r^2 + r^3 + r^4 + \cdots$$

$$-rS = -(r^2 + r^3 + r^4 + \cdots)$$

$$S - rS = r$$

$$S(1-r) = r$$

$$S = \frac{r}{1-r}$$

only applies when series converges.

Series converges ONLY when $|r|<1$

$$\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$$

sum(seq($\frac{1}{2^n}$,n,1,10))

$$\sum_{n=1}^{\infty}(\pi)^n = \frac{\pi}{1-\pi} \approx -1.467$$

$$\sum_{n=1}^{\infty}(1)^n = \frac{1}{1-1} =? \frac{1}{0}$$

Undefined

⇓

~~PS~~ diverges

lim = +∞

Feb 29-8:31 AM

10.2/4) a) $1 \cdot 2$, $1 \cdot 2 \cdot 3 \cdot 4$, $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6$,
$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8, \ldots$

1, 2, 3, 4

$(2n)!$

b) 1, $1 \cdot 2 \cdot 3$, $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$,
$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$

$(2n-1)!$

5) $\left\{ \frac{n}{n+2} \right\}_{n=1}^{\infty} = \frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \ldots$

$$\lim_{n\to\infty} \frac{n}{n+2} = \lim_{n\to\infty} \frac{n\,(1)}{n\left(1+\frac{2}{n}\right)} = \frac{1}{1+0} = \underline{1}$$

$\therefore$ converges

Feb 29-11:33 AM

A sequence is said to converge if $\lim_{n\to\infty} \{a_n\} = L$

If a $\lim_{n\to\infty} \{a_n\}$ does not exist, the sequence is said to diverge

10)

$$\left\{ n\sin\left(\frac{\pi}{n}\right) \right\}_{n=1}^{\infty}$$

$y = x\sin\frac{\pi}{x}$

$$\lim_{n\to\infty} n\sin\left(\frac{\pi}{n}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{\pi}{n}\right)}{\frac{1}{n}}$$

$\infty \cdot 0$

$$\text{"LH"} \lim_{n\to\infty} \frac{\pi\cos\left(\frac{\pi}{n}\right)\left(-\frac{1}{n^2}\right)}{-\frac{1}{n^2}} = \lim_{n\to\infty} \pi\cos\left(\frac{\pi}{n}\right) = \pi$$

Feb 29-7:52 AM

10.2/26b

$(-1)^{n+1} r^n$

$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ r, & -r^2, & r^3, & -r^4, \ldots \\ 0 & 1 & 2 & 3 \end{array}$

$(-1)^n r^{(n+1)}$

Serious about series :)

A series is a sum of the terms of a sequence.

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

A series converges to a sum L if the sequence of partial sums converges to L.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

$$\frac{n-1}{n}=\frac{n}{n}-\frac{1}{n}$$

$S_1=\frac{1}{2}$ $\quad 1-\frac{1}{2}$

$S_2=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ $\quad 1-\frac{1}{4}$

$S_3=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{7}{8}$ $\quad 1-\frac{1}{8}$

$S_4=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}=\frac{15}{16}$ $\quad 1-\frac{1}{16}$

$$\sum_{k=1}^{\infty}\frac{1}{2^k}$$

$$\left\{1-\frac{1}{2^n}\right\}_{n=1}^{\infty}$$

$$\lim_{n\to\infty} 1-\frac{1}{2^n}=1-0=1$$

sum(seq($\frac{1}{2^x}$, X, 1, 10))

3

LIST ⇒ 5

LIST ⇒ ⇒5

Geometric Series

A geometric series is a series where each term is r times the previous term.

r is called the common ratio

$\{ar^n\}$ is a geometric series

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$ geometric w/ $r = \frac{1}{2}$

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

$$-\left(\frac{1}{2}S\right) = -\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots\right)$$

$$\frac{1}{2}S = \frac{1}{2} + 0 + 0 + 0 + 0 + \cdots$$

$\therefore$ $\frac{1}{2}S = \frac{1}{2}$ $\therefore$ $S = 1$

$$\sum_{n=1}^{\infty} 3\left(\frac{1}{10^n}\right) = 3\sum_{n=1}^{\infty}\left(\frac{1}{10^n}\right)$$

$$\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\cdots = .333333333\overline{3}$$

let

$$S = \frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\frac{1}{10000}+\cdots$$

$$-\left(\frac{1}{10}S\right) = -\left(\frac{1}{100}+\frac{1}{1000}+\frac{1}{10000}+\cdots\right)$$

$$S-\frac{1}{10}S \qquad 0 \quad 0 \quad 0 \quad 0$$

$$= S\left(1-\frac{1}{10}\right) = \frac{9}{10}S = \frac{1}{10}$$

$$S = \frac{1}{10}\cdot\frac{10}{9} = \frac{1}{9}$$

$$3\sum_{n=1}^{\infty}\frac{1}{10^n} = 3\left(\frac{1}{9}\right) = \frac{1}{3}$$

Geometric Series ...... generally.

$$\sum_{n=1}^{\infty} ar^n = a\sum_{n=1}^{\infty} r^n$$

$$S = r + r^2 + r^3 + r^4 + r^5 + \cdots$$

$$-rS = -(r^2 + r^3 + r^4 + r^5 + \cdots)$$

$$S - rS = r \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$S(1-r) = r$$

$$S = \frac{r}{1-r} \quad \text{true}$$

if $\sum r^n$ converges.

Geometric Series converge if $|r| < 1$

$$\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$\sum_{n=1}^{\infty}(\pi)^n = \frac{\pi}{1-\pi} \approx -1.467$$

$$\sum_{n=1}^{\infty}(1)^n = 1+1+1+1+1 \cdots \cdots$$

$$\frac{1}{1-1} = \frac{1}{0} \quad \approx \infty$$